AP® Calculus AB (Operational)
2004 Sample Student Responses

A1

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$\int_0^{30} [82 + 4\sin(\frac{t}{2})]\,dt \approx 2474$ cars

2474 cars pass through over the 30-minute period.

Work for problem 1(b)

$F(t) = 82 + 4\sin(\frac{t}{2})$

$F'(t) = 4 \cdot \frac{1}{2} \cdot \cos(\frac{t}{2})$

$= 2\cos(\frac{t}{2})$

$F'(7) = 2\cos(\frac{7}{2}) = -1.8729$

Since the derivative of $F(t)$ is negative, the traffic flow is decreasing.

Do not write beyond this border.

Do not write beyond this border.

Continue problem 1 on page 5.

A₂

Work for problem 1(c)

Average value theo: $\dfrac{\int_a^b f(x)\,dx}{b-a}$

$$\frac{\int_{10}^{15}\left[82+4\sin\left(\frac{t}{2}\right)\right]dt}{15-10} = \frac{409.4962}{5} = 81.8992 \approx 82 \text{ cars/}\sout{\text{hour}}\text{ min}$$

Work for problem 1(d)

$$\frac{\int_{10}^{15} F'(t)\,dt}{15-10}$$

$$= \frac{\int_{10}^{15}\left[2\cos\left(\frac{t}{2}\right)\right]dt}{5}$$

$$= \frac{7.587697}{5}$$

$$= 1.5175$$

$$\approx 2 \text{ cars/min}^2$$

Do not write beyond this border.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

D

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$\int_0^{30} \left[82+4\sin\left(\frac{t}{2}\right)\right]dt$

2474 cars

Work for problem 1(b)

$F(7) = 82 + 4\sin(7/2)$

$F(7) = 83.403$

Traffic flow is increasing because F(7) is positive.

Do not write beyond this border.

Do not write beyond this border.

Continue problem 1 on page 5.

D2

Do not write beyond this border.

Work for problem 1(c)

$$\frac{1}{5}\int_{10}^{15}\left[82+4\sin\left(\frac{t}{2}\right)\right]dt =$$

81.899 cars

Work for problem 1(d)

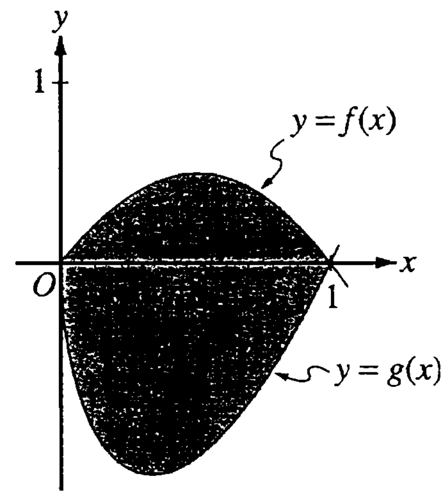
$$\frac{F(15)-F(10)}{15-10} = \frac{85.752-78.164}{5}$$

1.518 cars per minute

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

AP® Calculus AB (Operational)
2004 Sample Student Responses



Do not write beyond this border.

Work for problem 2(a)

$$\int_0^1 \left(2x(1-x)\right) - \left(3(x-1)\sqrt{x}\right) dx = 1.133$$

Do not write beyond this border.

Continue problem 2 on page 7.

Work for problem 2(b)

$$\pi\int_0^1 \left(2-3(x-1)\sqrt{x}\right)^2-\left(2-2x(1-x)\right)^2dx =$$

16.179

Do not write beyond this border.

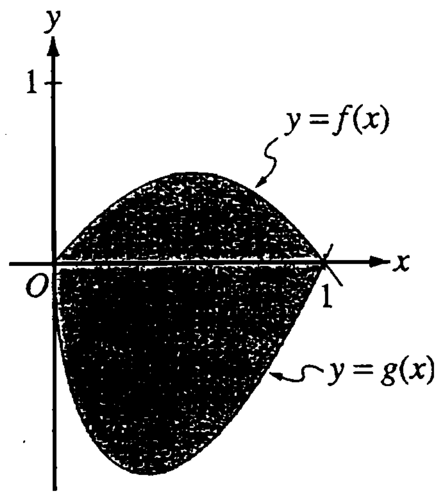
Do not write beyond this border.

Work for problem 2(c)

$h(x) = kx(1-x) \quad 0 \le x \le 1$

$$\int_0^1 \left[(kx(1-x))-(3(x-1)\sqrt{x})\right]^2dx = 15$$

GO ON TO THE NEXT PAGE.



$0 \le x \le 1$

$f(x) = 2x(1-x)$
$g(x) = 3(x-1)\sqrt{x}$

Work for problem 2(a)

Area under $f(x) = \int_0^1 2x(1-x)\,dx \approx .\overline{333} = \frac{1}{3}$

Area under $g(x) = \int_0^1 3(x-1)\sqrt{x}\,dx \approx -.8$

make Area under g(x) positive for ~~total~~ area

$|-.8| = .8$

Area enclosed by $f(x)$ & $g(x) = \frac{1}{3} + .8$

Area " " $= 17/15 \approx 1.1\overline{333}$ ✓

Do not write beyond this border.

Continue problem 2 on page 7.

Work for problem 2(b)

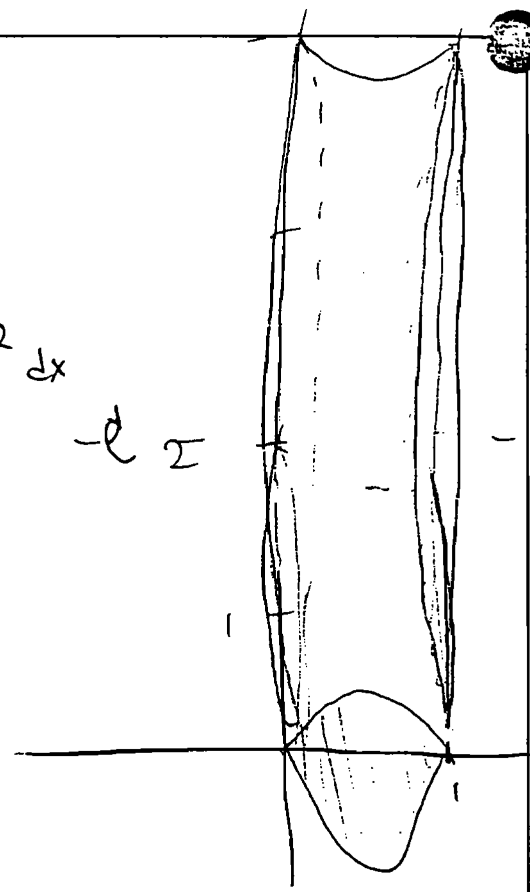
Volume of solid $\pi \int_0^1 R^2(x) - r^2(x)\, dx$

(outside function)
$R(x)$
$2 - g(x)$
$R(x) = 2 - 3(x-1)\sqrt{x}$

$\pi \int_0^1 (2-3(x-1)\sqrt{x})^2 - (2-2x(1-x))^2\, dx$

$= \frac{103\pi}{20} \approx 16.179$

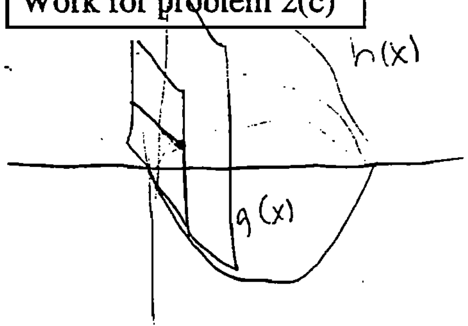
(inside function)
$r(x)$
$2 - f(x)$
$r(x) = 2 - 2x(1-x)$

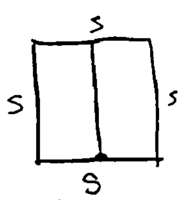


Work for problem 2(c)

$0 \le x \le 1$
$h(x) = kx(1-x)$

$k > 0$





$A_{\square} = s^2$

$s = h(x) - g(x)$

$A_{\text{me}\square} = \pi \int_0^1 kx(1-x) - 3(x-1)\sqrt{x}\, dx$

$16 = \pi \int_0^1 kx(1-x) - 3(x-1)\sqrt{x}\, dx$ ← use to find a 'k' value.

AP® Calculus AB (Operational)
2004 Sample Student Responses

Work for problem 3(a)

$a(t) = v'(t) = \frac{-e^t}{1+e^{2t}}$

$\therefore a(2) = \frac{-e^2}{1+e^4} = \boxed{-.133}$

Work for problem 3(b)

$v(2) = 1 - \tan^{-1}(e^2) = -.436$

speed is increasing because
at $t = 2$, $v(t) < 0$ and $a(t) < 0$

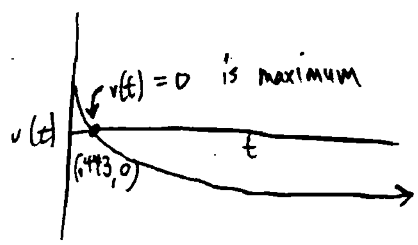
Do not write beyond this border.

Do not write beyond this border.

Continue problem 3 on page 9.

Work for problem 3(c)

$v(t) = 0 = 1 - \tan^{-1}(e^t)$



$\therefore$ $t = .443$ is a maximum
because $V(t) = y'(t) > 0$ on $0 \le t < .443$
and $v(t) = y'(t) < 0$ on $.443 < t < \infty$

Do not write beyond this border.

Work for problem 3(d)

$y(2) = -1 + \int_0^2 v(t) = -1 + \int_0^2 1 - \tan^{-1}(e^t) = \boxed{-1.361}$

$v(2) = 1 - \tan^{-1}(e^2) = -.436$

$\therefore$ particle is moving away from origin
because at $t = 2$ both $y(t)$ and $v(t)$ are less than zero

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Do not write beyond this border.

Work for problem 3(a)

$v(t) = 1 - \tan^{-1}(e^t)$

$a(t) = v'(t) = \dfrac{-1}{(e^t)^2+1} = \dfrac{1}{e^{2t}+1}$

$a(2) = \dfrac{1}{e^{2(2)}+1} = \dfrac{1}{e^4+1} \approx \boxed{.018}$

Work for problem 3(b)

$a(t) = \dfrac{1}{e^{2t}+1}$

$a(2) = .018$

The speed of the particle is increasing at time $t=2$ because $a(2)>0$, which means that the particle is accelerating at $t=2$.

Do not write beyond this border.

Continue problem 3 on page 9.

C

Work for problem 3(c)

(1) $v(t) = 1 - \tan^{-1}(e^t)$

$0 = 1 - \tan^{-1}(e^t)$

$1 = \tan^{-1}(e^t)$

$\tan(1) = e^t$

$\ln(\tan(1)) = t$

$.443 \approx t$

$v(.2) > 0$ $v(1) < 0$

⊕ ⊖

0 .443

increasing decreasing

(No other critical points)

The particle reaches its highest point at $t \approx .443$ because the velocity goes from positive to negative (graph stops increasing and starts decreasing), creating an absolute maximum.

Work for problem 3(d)

(a) $s(2) = s(0) + \int_0^2 v(t)\,dt$

$= (-1) + \int_0^2 (1 - \tan^{-1}(e^t))\,dt$

$\approx (-1) + (-.361)$

≈ -1.361 un

The particle is moving away from the origin at $t = 2$ because $v(2) < 0$. (see signchart 3(c))

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Do not write beyond this border.

Do not write beyond this border.

CollegeBoard
Advanced Placement Program

AP® Calculus AB (Operational)
2004 Sample Student Responses

NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$x^2 + 4y^2 = 7 + 3xy$

$2x + 4 \cdot 2y \cdot y' = 0 + 3xy' + 3y$

$8y \cdot y' - 3x \cdot y' = 3y - 2x$

$y'(8y-3x) = \frac{3y-2x}{8y-3x}$

$y' = \frac{3y-2x}{8y-3x}$ ✓

Do not write beyond this border.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$x^2 + 4y^2 = 7 + 3xy \quad x = 3 \quad y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$9 + 4y^2 = 7 + 9y \quad 4y^2 - 9y + 2 = 0$

$y = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 4}}{2 \cdot 4} = \frac{9 \pm \sqrt{81 - 32}}{8} = \frac{9 \pm \sqrt{49}}{8} = \frac{9 \pm 7}{8} = \frac{16}{8}, \frac{2}{8}$

$y = 2, \frac{1}{4}$

$\frac{3y - 2x}{8y - 3x} = 0 \quad 3y - 2x = 0 \quad 3y = 2x \quad 3y = 2 \cdot 3 \quad y = 2$ ✓

$P(3, 2)$

$\frac{3 \cdot 2 - 2 \cdot 3}{8 \cdot 2 - 3 \cdot 3} = \frac{0}{7}$ ✓ $= 0$

$16 - 9$

Work for problem 4(c)

$\frac{3y - 2x}{8y - 3x} = y'$

$8y \cdot y' - 3x \cdot y' = 3y - 2x$

$8y \cdot y'' + 8y' \cdot y' - 3x \cdot y'' - 3y' = 3y' - 2$

$y''(8y - 3x) = 3y' - 2 - 8y'^2 + 3y' =$

$y'' = \frac{3y' - 2 - 8y'^2 + 3y'}{8y - 3x}$

~~$y''(\ ) \to y'(2) =$~~ ~~$x = 3$~~ ~~$y = 2$~~

$y' = 0$
$x = 3$
$y = 2$

$y'' = \frac{0 - 2 - 0 + 0}{8 \cdot 2 - 3 \cdot 3} = \frac{-2}{16 - 9} = -\frac{2}{7} < 0$

$y'' < 0$ concave down

maximum

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$x^2 + 4y^2 = 7 + 3xy$

$\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$ show

$2x \cdot \frac{dx}{dx} + 8y\frac{dy}{dx} = 0 + 3\left(x \cdot \frac{dy}{dx} + y\right)$

$2x + 8y\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$

$8y\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - 2x$

$\frac{dy}{dx}(8y - 3x) = 3y - 2x$

$\frac{dy}{dx} = \boxed{\frac{3y-2x}{8y-3x}}$ ✓

Do not write beyond this border.

Continue problem 4 on page 11.

Work for problem 4(b)

$P(3, \underline{2})$

$m = 0$

$\frac{dy}{dx} = m = \frac{3y-2x}{8y-3x}$

$\frac{3y-2\cdot 3}{8y-3\cdot 3} = 0$

$3y-6=0$

$\frac{3y-6}{8y-9} = 0 \therefore \boxed{y=2}$

Work for problem 4(c)

$P(3,2)$ $\quad \frac{dy}{dx} = \frac{3y-2x}{8y-3x}$

$\frac{d^2y}{dx} = \frac{(8y-3x)(3\frac{dy}{dx}-2)-(3y-2x)(8\frac{dy}{dx}-3)}{(8y-3x)^2}$

$16 \quad -9 \qquad\qquad 0$

$\frac{d^2y}{dx} = \boxed{\frac{(8\cdot 2-3\cdot 3)(-2)-(3\cdot 2-2\cdot 3)(-3)}{(8\cdot 2-3\cdot 3)^2}}$

$\frac{d^2y}{dx} = \frac{-14-0}{49} = \left(-\frac{14}{9}\right)$

$\frac{3y-2x}{8y-3x}$ [illegible]

The curve has a local maximum because at pt. P(3,2), the 2nd derivative is negative. This makes the curve concave down.

Do not write beyond this border.

AP® Calculus AB (Operational)
2004 Sample Student Responses

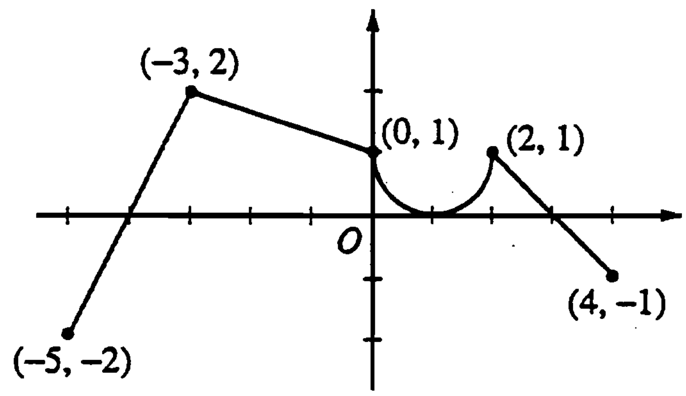
The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 4,500 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

For further information, visit www.collegeboard.com

For the College Board's online home for AP professionals, visit AP Central at apcentral.collegeboard.com.

B1

NO CALCULATOR ALLOWED.



Graph of f

Work for problem 5(a)

$g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(2+1)3 = \frac{1}{2}3\,3 = \frac{9}{2} = \boxed{4.5}$

$g'(0) = f(0) = \boxed{1}$

Work for problem 5(b)

$g'(x) = f(x)$

x		-4		1		3	
$g'(x)$ $f(x)$	$-$		$+$		$+$		$-$

relative maximum at $x=3$

At $x=3$, the slope of $g(x)$ changes from positive to negative

 B2

NO CALCULATOR ALLOWED

Do not write beyond this border.

Work for problem 5(c)

x	g(x)
-5	0
-4	-1
4	+

$\int_{-3}^{-5} f(t)\,dt = 0$

$\int_{-3}^{-4} f(t)\,dt = -1$

$\int_{-3}^{4} f(t)\,dt = +$

$g(-4) = \boxed{-1}$

at x=4, the slope of g(x) changes from negative to positive. The value of g(-4) is less than g(-5) or g(4).

Work for problem 5(d)

$g'(x) = f(x)$

$g''(x) = f'(x)$

x: (-3) (0) 1 (2)

g''(x), f'(x): + | − | − | + | −

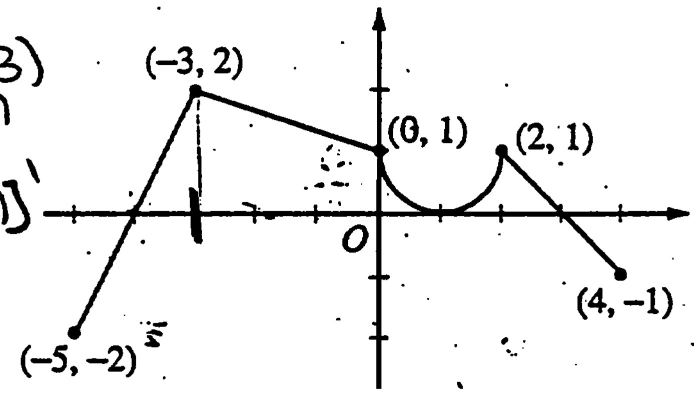
$\boxed{x = -3, 1, 2}$

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

$g(x) = F(x) - F(-3)$
$g'(x) = [F(x) - F(-3)]'$
$= F'(x) - [F(-3)]'$
$= F'(x) - 0$
$= f(x)$
$g'(x) = f(x)$



Graph of *f*

Work for problem 5(a)

$g(x) = \int_{-3}^{x} f(t)\,dt$

$g(0) = \int_{-3}^{0} f(t)\,dt$ = area of trapezoid $= \left(\frac{1}{2}\right)(2+1)(3) = \frac{1}{2}(3)(3) = \frac{9}{2}$

$g(0) = \frac{9}{2}$

$g'(0) = f(0) = 1$ $\quad g'(0) = 1$

Do not write beyond this border.

Work for problem 5(b)

g is a relative max on (−5, 4) $\quad g'(x) = f(x)$

g(x) has a local maximum where g'(x) is zero or where g'(x) is undefined, and where g'(x) changes sign from positive to negative. This occurs only at x = 3, where g'(x) = 0 and changes from positive to negative. [at x = 1, g'(x) does not experience a sign change; at x = −4, g'(x) changes from negative to positive, a local minimum]

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c) abs min on [-5,4]

There is a local min at $x=-4$, [on g(x)] because $g'(x)$ is equal to zero and changes from negative to positive.

$g(-4)=\int_{-3}^{-4} f(t)dt = -\int_{-4}^{-3} f(t)dt = -[\frac{1}{2}(2+1)(3)+\frac{1}{2}(1)(2)]$

$= -[\frac{9}{2}+1] = -\frac{11}{2} = -5.5$

$g(-4) = -\frac{11}{2}$

The absolute minimum of g on [-5,4] is at $x=-4$ because $g'(x)=0$ at $x=-4$ & changes from negative to positive. Also, the endpoints' values on g are bigger than the value of g(-4).

ENDPOINT CHECK

$g(-5) = \int_{-3}^{-5} f(t)dt = -\int_{-5}^{-3} f(t)dt = -[\frac{1}{2}(2+1)(3)+\frac{1}{2}(1)(2) - 1(2)(\frac{1}{2})]$

$= -[\frac{11}{2}-1] = -\frac{9}{2} = -4.5$

$g(-5) = -\frac{9}{2} > -\frac{11}{2}$

$g(4) = \int_{-3}^{4} f(t)dt = \frac{1}{2}(2+1)(3) + [(1)(2)-(\pi)(1^2)(\frac{1}{2})] + (1)(1)(\frac{1}{2}) - 1(1)(\frac{1}{2})$

$= \frac{9}{2} + [2-\frac{\pi}{2}] + 0$

$= \frac{9}{2}+2-\frac{\pi}{2} = \frac{13}{2}-\frac{\pi}{2} \approx 6.5-1.57 \rightarrow$ positive #

Work for problem 5(d) PI's on g(x) at $x=-3, 2,$ and $x=1$.

g has a point of inflection (PI) where $g''(x)$ is zero or undefined & changes signs. $g''(x)$ equals zero and changes signs at $x=1$, where $g'(x)$ is a local min. $g''(x)$ is undefined and changes signs at $x=-3$ and $x=2$ (even though $g''(x)$ is undefined at $x=0$, $g''(x)$ doesn't change signs there) — These are local max's.

GO ON TO THE NEXT PAGE.

Do not write beyond this border.

CollegeBoard
Advanced Placement Program

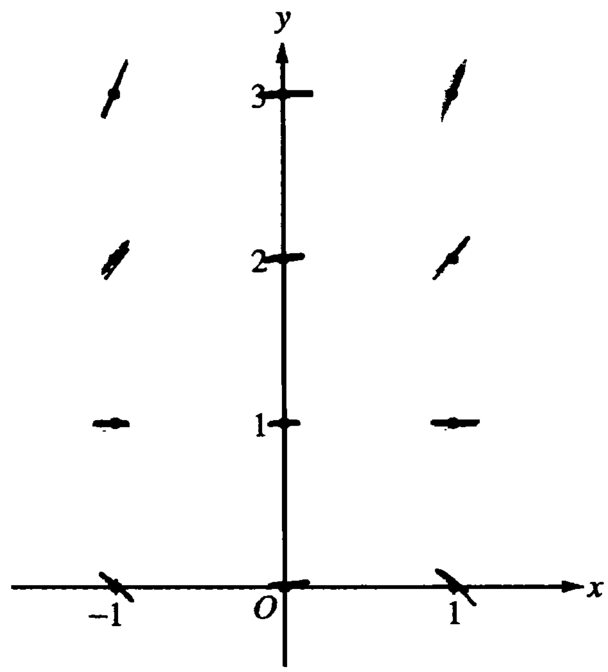
AP® Calculus AB (Operational) 2004 Sample Student Responses

A1

NO CALCULATOR ALLOWED

Do not write beyond this border.

Work for problem 6(a)



Work for problem 6(b)

$y > 1$;
$x \neq 0$;

Do not write beyond this border.

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$\int \frac{1}{y-1}\,dy = \int x^2\,dx$$

$$\ln|y-1| = \frac{1}{3}\cdot x^3 + C$$

$$y-1 = e^{x^3/3 + C}$$

$$y = ke^{x^3/3} + 1$$

$$f(x) = ke^{x^3/3} + 1$$

$$f(0) = 3 = k(1) + 1$$

$$k = 2$$

$$f(x) = 2e^{x^3/3} + 1$$

Do not write beyond this border.

Do not write beyond this border.

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.**

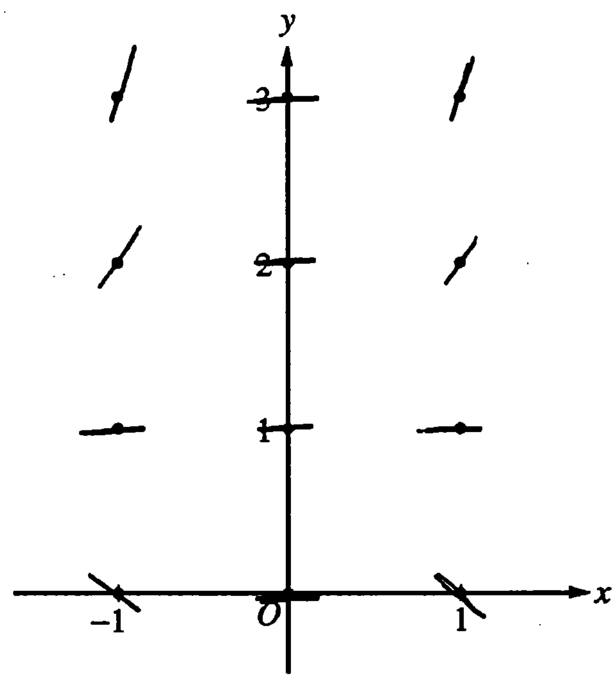
C1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$\frac{dy}{dx} = x^2(y-1)$

$(-1,0) = -1$
$(-1,1) = 0$
$(-1,2) = 1$
$(-1,3) = 2$
$(0,0) = 0$
$(0,1) = 0$
$(0,2) = 0$
$(0,3) = 0$
$(1,0) = -1$
$(1,1) = 0$
$(1,2) = 1$
$(1,3) = 2$



Work for problem 6(b)

b) if the y value of the point is $1<y$ then the slope will be positive.

Do not write beyond this border.

Continue problem 6 on page 15.

C2

NO CALCULATOR ALLOWED

Work for problem 6(c)

$\frac{dy}{dx} = x^2(y-1)$ $u = y-1$

$du = 1$

$x^2(y-1)dx = dy$

$\int x^2 dx = \int \frac{dy}{y-1}$

$\frac{x^3}{3} + c = \ln(y-1)$

$e^{1/3x^3 + c} = y-1$

$Ce^{1/3x^3} + 1 = y$ $f(0) = 3$

$Ce^{1/3(0)^3} + 1 = 3$

$Ce^0 = 2$

$C = 2$

$y = 2e^{(1/3x^3)} + 1$

Do not write beyond this border.

Do not write beyond this border.

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.**